<u>ΦΥΣΙΚΗ</u>

ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΥ

12 ΙΟΥΝΙΟΥ 2019

ΑΠΑΝΤΗΣΕΙΣ

ΘΕΜΑ Α

Α1. **β**

Α2. **γ**

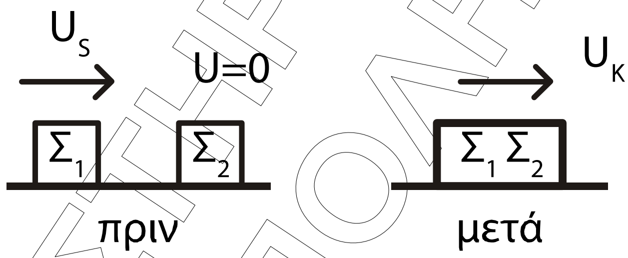
Α3. **α**

Α4. **γ**

Α5. **α) → Λ,** **β) → Σ,** **γ) → Λ,** **δ) → Σ,** **ε) → Σ**

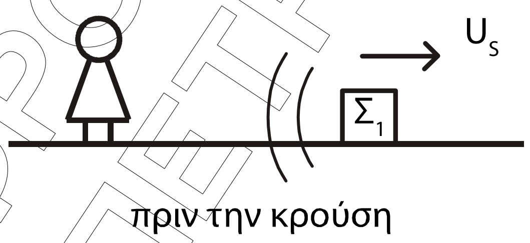
ΘΕΜΑ Β

Β1. Πλαστική κρούση $Σ_1 – Σ_2$.



<u>Α.Δ.Ο.</u> $\vec{P}_{πριν} = \vec{P}_{μετά}$

$$\Rightarrow m\upsilon_s + 0 = (m+m)\upsilon_k \Rightarrow \left.\begin{array}{l} \upsilon_k = \dfrac{\upsilon_s}{2} \\ \upsilon_s = \dfrac{\upsilon_{ηχ}}{20} \end{array}\right\} \dfrac{\upsilon_{ηχ}}{40} \quad (1)$$



$$f_1 = \frac{\upsilon_{ηχ} + 0}{\upsilon_{ηχ} + \upsilon_s} f_s = \frac{\upsilon_{ηχ}}{\upsilon_{ηχ} + \upsilon_s} f_s \quad (2)$$

μετά την κρούση

$$f_2 = \frac{\upsilon_{\eta\chi}}{\upsilon_{\eta\chi} + \upsilon_k} f_s = \frac{\upsilon_{\eta\chi}}{\upsilon_{\eta\chi} + \frac{\upsilon_s}{2}} f_s \quad (3)$$

Από $\frac{(2)}{(3)} \Rightarrow \frac{f_1}{f_2} = \frac{\frac{\upsilon_{\eta\chi}}{\upsilon_{\eta\chi}+\upsilon_s} f_s}{\frac{\upsilon_{\eta\chi}}{\upsilon_{\eta\chi}+\upsilon_k} f_s} = \frac{\upsilon_{\eta\chi}+\upsilon_k}{\upsilon_{\eta\chi}+\upsilon_s} =$

$$= \frac{\upsilon_{\eta\chi} + \frac{\upsilon_{\eta\chi}}{40}}{\upsilon_{\eta\chi} + \frac{\upsilon_{\eta\chi}}{20}} = \frac{\frac{41}{40}}{\frac{21}{20}} = \frac{41}{42}$$

Άρα σωστό το (ii)

B2. Εξ. συνέχ. Από $\Delta \rightarrow \Gamma$

$\Pi_1 = \Pi_2 \Rightarrow A_1\upsilon_1 = A_2\upsilon_2 \Rightarrow 2A_2 \cdot \upsilon_1 = A_2\upsilon_2 \Rightarrow \underline{\upsilon_2 = 2\upsilon_1} \quad (1)$

Bernoulli: $\Delta \rightarrow \Gamma$

$$\left.\begin{array}{c} P_\Delta + \frac{1}{2}\rho\upsilon_1^2 = P_2 + \frac{1}{2}\rho\upsilon_2^2 \\ \text{Στον κατακόρυφο σωλήνα} \\ P_\Delta = P_{atm} + g\rho h \end{array}\right\} \Rightarrow P_{atm} + \rho g h + \frac{1}{2}\rho\upsilon_1^2 = P_{atm} + \frac{1}{2}\rho\upsilon_2^2 \Rightarrow$$

$$\Rightarrow gh + \frac{1}{2}\upsilon_1^2 = \frac{1}{2}\upsilon_2^2 \overset{(1)}{\Rightarrow} gh + \frac{1}{2}\frac{\upsilon_2^2}{4} = \frac{1}{2}\upsilon_2^2 \Rightarrow$$

$$\Rightarrow \frac{3}{8}\upsilon_2^2 = gh \Rightarrow \underline{\upsilon = \sqrt{\frac{8}{3}gh}} \quad (2)$$

$$\left.\begin{array}{r} \Pi_2 = \Pi_3 \\ \text{Στο δοχείο η επιφάνεια σταθερή σε ύψος (H)} \\ \text{άρα:} \end{array}\right\} \Rightarrow A_2\upsilon_2 = A_3\upsilon_3 \Rightarrow$$

$$\Rightarrow A_2\upsilon_2 = \frac{A_2}{2}\upsilon_3 \Rightarrow \underline{\upsilon_3 = 2\upsilon_2}$$

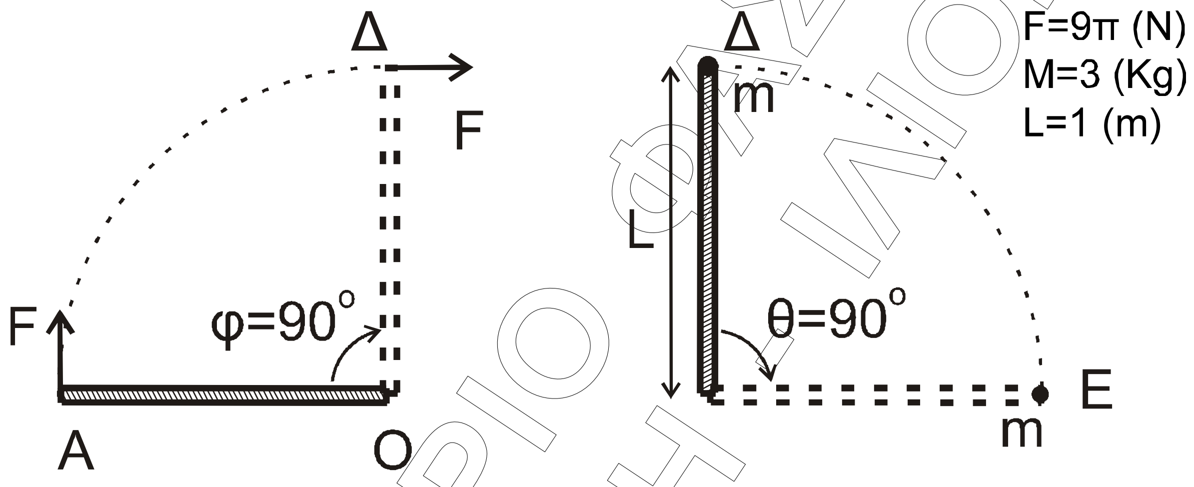
Bernoulli: $E \rightarrow Z$

$$P_{atm} + \rho g H + 0 = P_{atm} + \frac{1}{2}\rho \upsilon_3^2 + 0 \Rightarrow gH = \frac{1}{2}4\upsilon_2^2 \Rightarrow$$

$$\Rightarrow gH = 2\upsilon_2^2 \Rightarrow \upsilon_2 = \sqrt{g\frac{H}{2}} \quad \text{από την (2)} \Rightarrow \sqrt{\frac{8}{3}gh} = \sqrt{g\frac{H}{2}} \Rightarrow \frac{h}{H} = \frac{3}{16}$$

Σωστό (iii)

B3.



Σχήμα 4 Σχήμα 5

Για την κίνηση $A \rightarrow \Delta$ από το Θ.Μ.Κ.Ε. ισχύει:

$$\Delta K = \Sigma W \Rightarrow \frac{1}{2}\cdot I_O \cdot \omega_\Delta^2 = (F \cdot L)\frac{\pi}{2} \Rightarrow \frac{1}{2}\cdot\frac{1}{3}\cdot M \cdot L^2 \cdot \omega_\Delta^2 = F \cdot L \cdot \frac{\pi}{2} \Rightarrow$$

$$\Rightarrow \frac{1}{2}\cdot\frac{1}{3}\cdot 3 \cdot 1 \cdot \omega_\Delta^2 = 9\pi \cdot 1 \cdot \frac{\pi}{2} \Rightarrow \omega_\Delta = 3\pi \ \ rad/s$$

Από Α.Δ.Σ. στην κρούση στο (Δ) ισχύει:

$$\vec{L}_{\pi\rho\iota\nu} = \vec{L}_{\mu\varepsilon\tau\acute{\alpha}} \Rightarrow I_O \cdot \omega_\Delta = I'_O \cdot \omega'_\Delta \Rightarrow \omega'_\Delta = \frac{I_O \cdot \omega_\Delta}{I'_O} \qquad (1)$$

Όμως $I'_O = \frac{1}{3}M \cdot L^2 + mL^2 = \frac{3\cdot 1^2}{3} + 1\cdot 1^2 = 2 \ Kgm^2 \quad (2)$

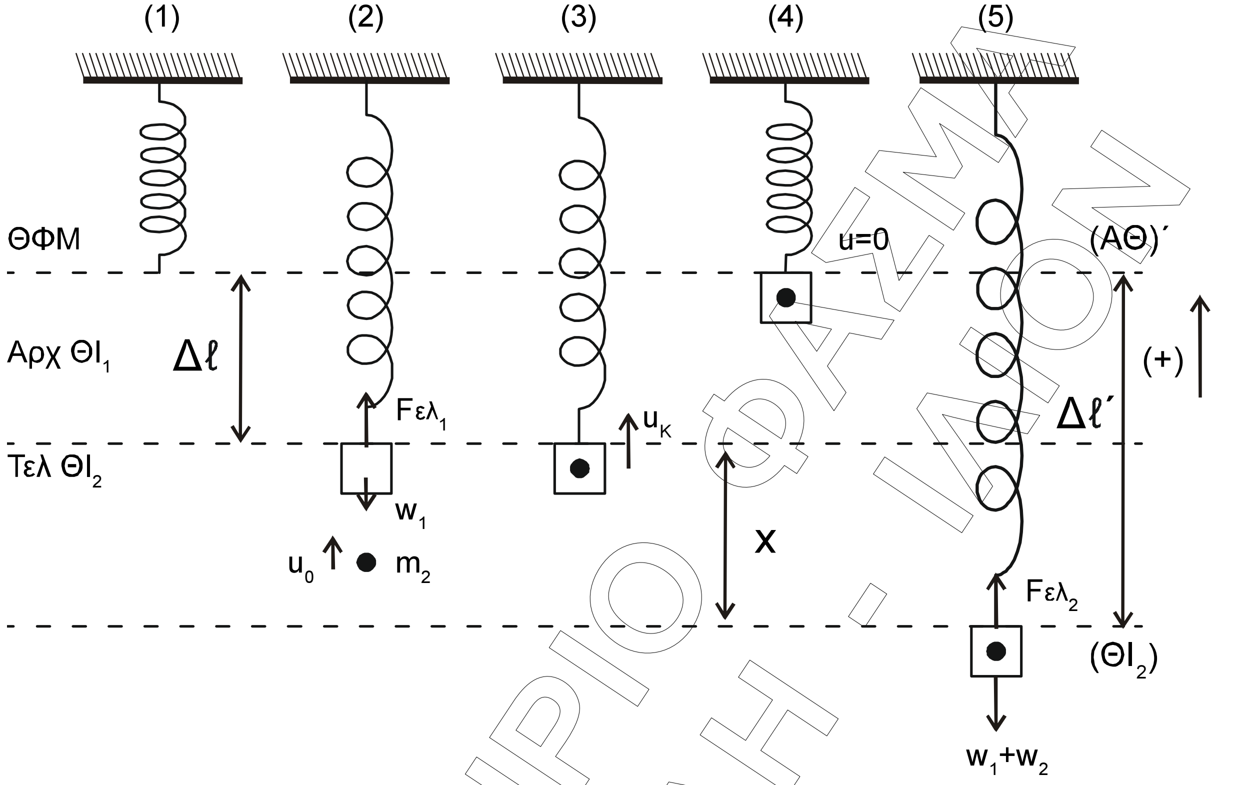
Από (1) και (2) έχουμε: $\omega'_\Delta = \dfrac{\frac{ML^2}{3}\cdot\omega_\Delta}{2} = \dfrac{\frac{3\cdot 1}{3}\cdot 3\pi}{2} = \dfrac{3\pi}{2} \ \ rad/s$

Για τον χρόνο $t_{\Delta\rightarrow E} = t$ έχουμε

$$\Delta\Theta = \omega'_\Delta \cdot t \Rightarrow \frac{\pi}{2} = \frac{3\pi}{2}\cdot t \Rightarrow t = \frac{1}{3} \ (s)$$

Άρα σωστό είναι το (ii).

ΘΕΜΑ Γ



Γ1. Για την αρχική Θ.Ι$_1$ σχήμα (2) ισχύει:

$$\Sigma F_y = 0 \Rightarrow W_1 = F_{\varepsilon\lambda.1} \Rightarrow m_1 \cdot g = \mathrm{K} \cdot \Delta l \Rightarrow \mathrm{K} = \frac{10}{0,05} = 200 \ \ \mathrm{N}/m$$

Για την τελική ΘΙ$_2$, σχήμα (5)

$$\Sigma F_y = 0 \Rightarrow W_1 + W_2 = F_{\varepsilon\lambda 2} \Rightarrow (m_1 + m_2) \cdot g = \mathrm{K} \cdot \Delta l' \Rightarrow 20 = 200 \cdot \Delta l' \Rightarrow \Delta l' = 0,1 \ m$$

Άρα το πλάτος ΑΘ'-ΘΙ$_2$: $\Delta l' = \mathrm{A} = 0,1 \ m$

Γ2. $x = \Delta l' - \Delta l = 0,05 \ m$

ΑΔΕΤ:

$$\mathrm{K} + U = \mathrm{E_T} \Rightarrow \frac{1}{2} \cdot (m_1 + m_2) \cdot \upsilon_{\mathrm{K}}^2 + \frac{1}{2} \cdot k \cdot x^2 = \frac{1}{2} \cdot k \cdot \mathrm{A}^2 \Rightarrow 2 \cdot \upsilon_{\mathrm{K}}^2 + 200 \cdot 0,05^2 = 200 \cdot 0,1^2 \Rightarrow$$

$$\Rightarrow \upsilon_{\mathrm{K}}^2 = 1 - 0,25 \Rightarrow \upsilon_{\mathrm{K}} = \sqrt{0,75} = \frac{\sqrt{3}}{2} \ m/s$$

$$\text{Από}: \ \overrightarrow{P_{\pi\rho\iota\nu}} = \overrightarrow{P_{\mu\varepsilon\tau\acute{\alpha}}} \Rightarrow m \cdot \upsilon_o = 2 \cdot m \cdot \upsilon_K \Rightarrow \upsilon_o = \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3} \ m/s$$

$$\text{Άρα} \ \ \mathrm{K} = \frac{1}{2} \cdot m \cdot \upsilon_o^2 = \frac{1}{2} \cdot 1 \cdot \sqrt{3}^2 = \frac{3}{2} = 1,5 \ J$$

Γ3.

$$\Delta\overrightarrow{P_2} = \overrightarrow{P_2'} - \overrightarrow{P_2} = \Delta P_2 = m_2\upsilon_k - m_2\upsilon_0 \Rightarrow$$

$$\Delta P_2 = \frac{\sqrt{3}}{2} - \sqrt{3} \Rightarrow \Delta P_2 = -\frac{\sqrt{3}}{2}\ \text{kg}\,{}^{\text{m}}\!/_{\text{s}} \Rightarrow$$

$$\Rightarrow |\Delta \text{P}_2| = \frac{\sqrt{3}}{2}\ \text{kg}\,{}^{\text{m}}\!/_{\text{s}}$$

Με κατεύθυνση προς τα κάτω, προς τα αρνητικά.

Γ4. Για $t = 0$, $x = 0{,}05\text{m}$, $A = 0{,}1\ \text{m}$, $\upsilon > 0$

$$x = A\eta\mu(\omega t + \varphi_0) \overset{t=0}{\Rightarrow} 0{,}05 = 0{,}1\ \eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0 = \frac{1}{2} = \eta\mu\frac{\pi}{6}$$

$$\left.\begin{array}{l}\text{άρα}\quad \varphi_0 = 2\kappa\pi + {}^{\pi}\!/_{6} \\ \qquad\qquad\qquad 0 \le \varphi_0 < 2\pi \\ \text{ή}\quad \varphi_0 = 2\kappa\pi + {}^{5\pi}\!/_{6}\end{array}\right\} \overset{\kappa=0}{\Rightarrow}$$

$$\left.\begin{array}{l}\varphi_0 = \dfrac{\pi}{6}\ \text{με}\ \ \upsilon > 0\ \ \text{δεκτή} \\ \varphi_0 = \dfrac{5\pi}{6}\qquad \upsilon < 0\end{array}\right\} \text{άρα}\ \varphi_0 = \frac{\pi}{6}$$

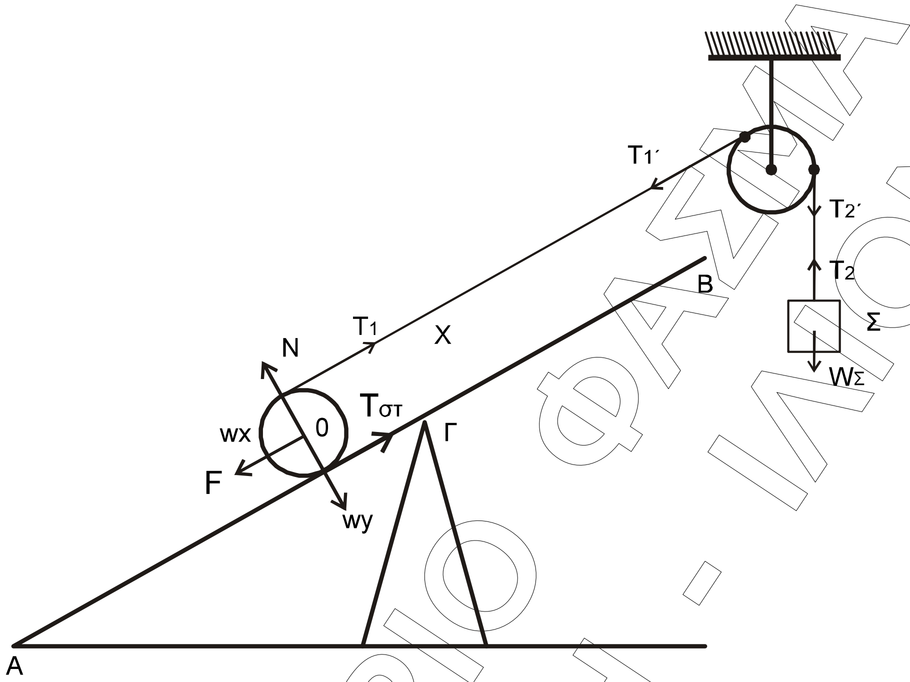
$$w = \sqrt{\frac{k}{m_1 + m_2}} = \sqrt{100} = 10\ {}^{\text{rad}}\!/_{\text{s}}$$

Άρα η απομάκρυνση είναι

$$x = 0{,}1\eta\mu\left(10t + \frac{\pi}{6}\right)\ \text{(SI)}$$

ΘΕΜΑ Δ

Δ1.



Σώμα (Σ):

$$\left.\begin{aligned}\Sigma F y = 0 \Rightarrow T_2 = W_\Sigma \\ T_2' = T_2\end{aligned}\right\} T_2' = W_\Sigma \Rightarrow T_2' = 20\,\text{N}$$

Στην τροχαλία:

$$\Sigma\tau_{(K)} = 0 \Rightarrow T_1' R_T = T_2' R_T \Rightarrow T_1' = T_2' \Rightarrow T_1' = 20\,\text{N}$$

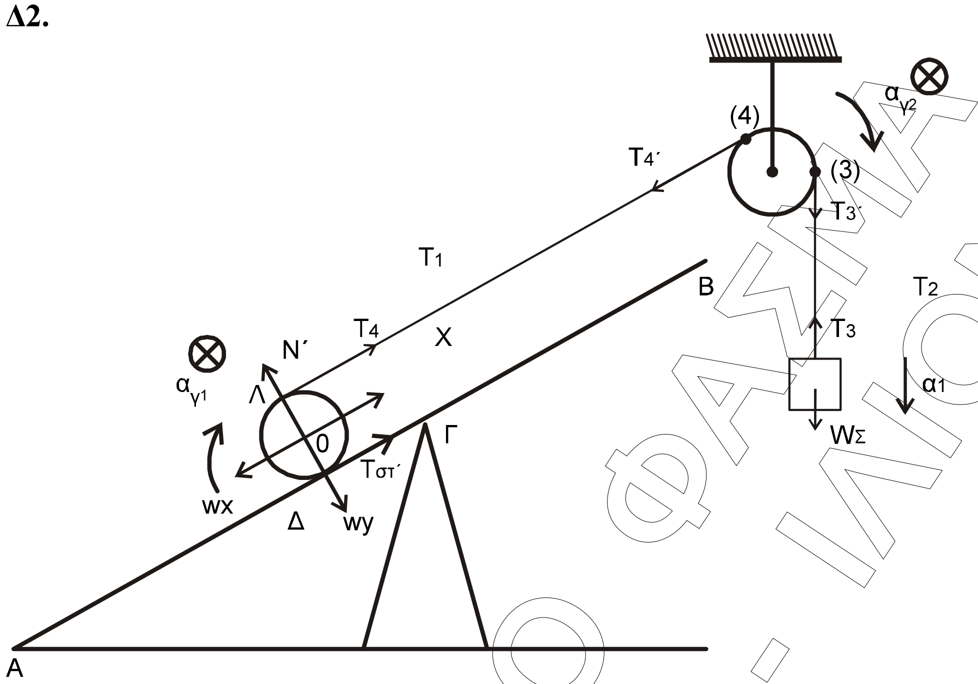
$$T_1 = T_1' = 20\,\text{N}$$

Στον κύλινδρο:

$$\Sigma\tau_{(0)} = 0 \Rightarrow T_1' R_K = T_{\sigma\tau.} R_K \Rightarrow T_{\sigma\tau.} = 20\,\text{N}$$

$$\Sigma F_x = 0 \Rightarrow F + W_x = T_1 + T_{\sigma\tau.} \Rightarrow F = 40 - 10 \Rightarrow \underline{F = 30\,\text{N}}$$

Δ2.



Για τον κύλινδρο $T_3 = T_3'$, $T_4 = T_4'$ αβαρή νήματα

μεταφ: $\Sigma F_x = m_\kappa \cdot a_{cm} \Rightarrow T_4 + T_{\sigma\tau}' - W_x = m_\kappa \cdot a_{cm}$ (1)

στροφ: $\Sigma \tau_{(0)} = \mathrm{I}_0 \cdot \alpha_{\gamma_1} \Rightarrow T_4 \cdot R_k - T_{\sigma\tau}' \cdot R_\kappa = m_\kappa \dfrac{R_\kappa^2}{2} \alpha_{\gamma_1}$ (2)

<u>Στην τροχαλία:</u>

στροφ: $\Sigma \tau_\kappa = \mathrm{I}_\kappa \cdot \alpha_{\gamma_2} \Rightarrow T_4' \cdot R_T - T_3' \cdot R_T = m_T \dfrac{R_T^2}{2} \alpha_{\gamma_2}$ (3)

<u>Στο σώμα:</u>

μεταφ: $\Sigma F_y = m_\Sigma \cdot a \Rightarrow W_\Sigma - T_3 = m_\Sigma \cdot a$ (4)

η ταχύτητα του σώματος $\upsilon_\Sigma = \upsilon_3$ άρα $\alpha_2 = \alpha \Rightarrow \alpha = \alpha_{\gamma_2} R_T$ (5)

$$\left.\begin{array}{l} \text{η ταχύτητα } \upsilon_\Lambda = \upsilon_4 \text{ άρα } \alpha_\Lambda = \alpha_4 = \alpha_{\gamma_2} \cdot R_T \\ \text{όμως } \upsilon_\Lambda = 2\upsilon_{cm} \text{ άρα } \alpha_\Lambda = 2a_{cm} \\ \upsilon_\Delta = 0 \text{ άρα } \upsilon_{cm} = \omega \cdot R_\kappa \text{ άρα } \alpha_{cm} = a_{\gamma_1} R_\kappa \quad (7) \\ 2a_{cm} = \alpha \ (6) \end{array}\right\} \Rightarrow$$

<u>Λύνοντας:</u>

Από (2),(7) $\Rightarrow \left(T_4' - T_{\sigma\tau}'\right) R_\kappa = \dfrac{m_\kappa R_\kappa^2}{2} \dfrac{\alpha_{cm}}{R_\kappa} \Rightarrow T_4' - T_{\sigma\tau} = \dfrac{m_\kappa}{2} \alpha_{cm} \overset{(m_\kappa = 2)}{\Rightarrow} T_4' - T_{\sigma\tau} = \alpha_{cm}$ (8)

Από (3),(5) $\Rightarrow \left(T_3 - T_4\right) R_T = \dfrac{m_T R_T^2}{2} \dfrac{\alpha}{R_T} \Rightarrow T_3 - T_4 = \dfrac{m_T}{2} \alpha \overset{(m_T = 2)}{\Rightarrow} T_3 - T_4 = \alpha$ (9)

Από (8),(6) $\Rightarrow \qquad T_4 - T_{\sigma\tau} = \dfrac{\alpha}{2}$ (10)

$$\left.\begin{array}{ll} (4) & 20 - T_3 = 2\alpha \\ (9) & T_3 - T_4 = \alpha \end{array}\right\} 20 - T_4 = 3\alpha \quad (11)$$

$A\pi\acute{o}\,(1)(6) \Rightarrow \quad T_4 + T_{\sigma\tau} - m_\kappa g \eta\mu\varphi = m_\kappa \cdot a_{cm}$

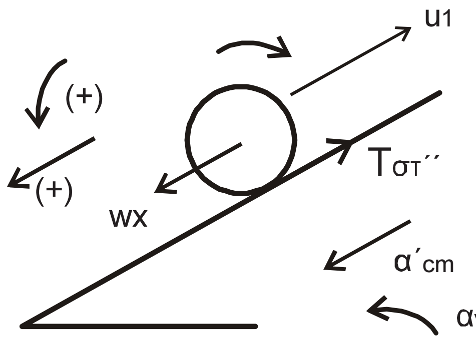
$$T_4 + T_{\sigma\tau} - 2\cdot 10\frac{1}{2} = 2\frac{a}{2} \Rightarrow T_4 - T_{\sigma\tau} = 10 + a \quad (12)$$

$A\pi\acute{o}\,(10),(12) \;\Rightarrow 2T_4 = 10 + \frac{3\alpha}{2} \overset{(11)}{\Rightarrow}$

$$2\left(20-3a\right) = 10 + \frac{3a}{2} \Rightarrow 40 - 6a = 10 + \frac{3a}{2} \Rightarrow a = 4\ {}^{m}\!/\!_{s^2}$$

(6) $\qquad a_{cm} = \frac{\alpha}{2} = 2\ {}^{m}\!/\!_{s^2}$.

Δ3.



$\upsilon_1 = \alpha_1 \cdot t = 2\cdot 0,5 = 1\ m/s$

$\Sigma F_x = M\cdot \alpha_{cm}$

$W_x - Ts'' = M_K \cdot \alpha_{cm}'$

$M_K \cdot g \cdot \eta\mu\,\varphi - Ts'' = M_K \cdot \alpha_{cm}' \quad (1)$

$\Sigma\tau = I_K \cdot \alpha'_{\gamma\omega\nu} \Rightarrow T_s'' \cdot R_K = \frac{M_K R_K^2}{2} \cdot \alpha'_{\gamma\omega\nu} \quad (2)$

κύλιση $\alpha'_{cm} = \alpha'_{\gamma\omega\nu} \cdot R_K \quad (3)$

Από (2) και (3) $\quad T_s'' \cdot R_K = M_K \frac{R_K^2}{2} \cdot \frac{\alpha'_{cm}}{R_K} \Rightarrow T_s'' = \frac{M_K \cdot \alpha'_{cm}}{2} = \frac{2\cdot\alpha'_{cm}}{2} \Rightarrow$

$\Rightarrow T_s'' = \alpha'_{cm} \quad (4)$

(1), (4) $\quad 2\cdot 10\cdot\frac{1}{2} - \alpha_{cm}' = 2\alpha'_{cm} \Rightarrow 10 = 3\alpha'_{cm} \Rightarrow$

$\Rightarrow \alpha'_{cm} = \frac{10}{3}\ m/s^2$

$\upsilon = \upsilon_1 - \alpha_{cm} \cdot \Delta t \Rightarrow 0 = 1 - \frac{10}{3}\cdot\Delta t \Rightarrow \Delta t = \frac{3}{10} = 0,3\ s$

$t_{STOP} = 0,5 + \Delta t = 0,8\ s$

Δ4.

$$S_{ΟΛ} = S_1 + S_2$$

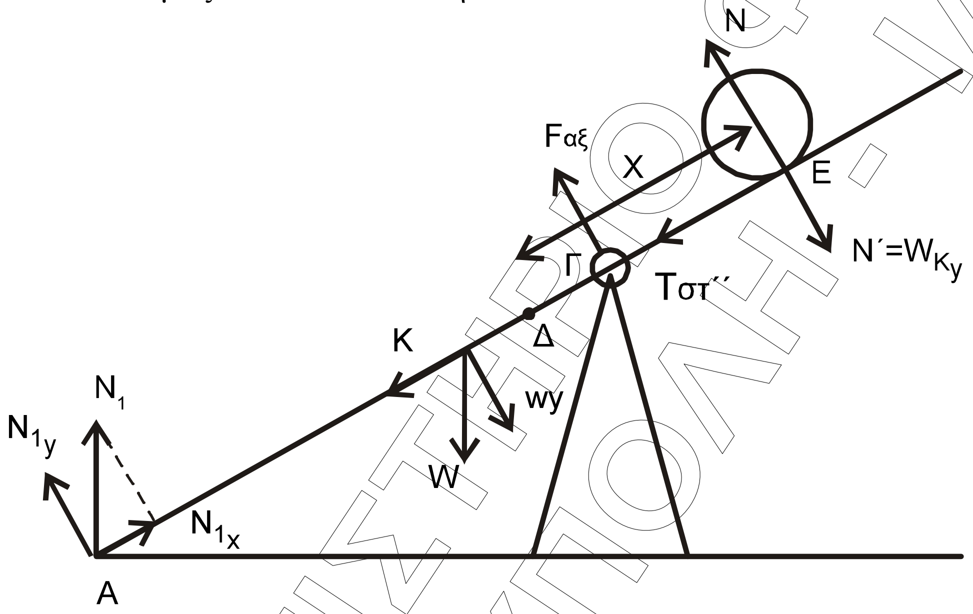
$$S_1 = \frac{1}{2}\alpha_1 t_1{}^2 = \frac{1}{2}\cdot 2\cdot 0,5^2 = 0,25\ \text{m}$$

$$S_2 = \upsilon_1 \cdot \Delta t - \frac{1}{2}\cdot \alpha_{cm}\cdot \Delta t^2 = 1\cdot 0,3 - \frac{1}{2}\cdot\frac{10}{3}\cdot 0,3^2 =$$

$$= 0,3 - \frac{1}{2}\cdot\frac{10}{3}\cdot 0,3\cdot 0,3 = 0,3 - \frac{1}{2}\cdot 0,3 = 0,15\ \text{m}$$

$$S_{ΟΛ} = 0,25 + 0,15 = 0,4\ \text{m}$$

Δ5. Οι δυνάμεις που ασκούνται στην σανίδα



$$\Sigma\tau_{(\Gamma)} = 0 \Rightarrow W_y(\text{ΚΓ}) - \text{N}_{1y}(\text{ΑΓ}) - W_{K_y}(\text{ΓΕ}) = 0 \Rightarrow$$

$$\Rightarrow \text{M}g\cdot\sigma\upsilon\nu\varphi(\text{ΚΓ}) - \text{M}_\text{K}\cdot g\cdot\sigma\upsilon\nu\varphi(\text{ΓΕ}) = \text{N}_1\cdot\sigma\upsilon\nu\varphi(\text{ΑΓ}) \Rightarrow$$

$$\Rightarrow 20\cdot 0,5 - 20\cdot 0,2 = \text{N}_1\cdot 2,5 \Rightarrow \text{N}_1 = \frac{6}{2,5} = 2,4\ \text{N}$$

Εναλλακτικά

$$\Sigma\tau_{(\Gamma)} = 0 \Rightarrow W_y(\text{ΚΓ}) - \text{N}_{1y}(\text{ΑΓ}) - W_{K_y}(x - \text{ΓΔ}) = 0 \Rightarrow$$

$$\Rightarrow \text{M}g\cdot\sigma\upsilon\nu\varphi(\text{ΚΓ}) - \text{M}_\text{K}\cdot g\cdot\sigma\upsilon\nu\varphi(x - \text{ΓΔ}) = \text{N}_1\cdot\sigma\upsilon\nu\varphi(\text{ΑΓ}) \Rightarrow$$

$$\Rightarrow 20\cdot 0,5 - 20\cdot(x - 0,2) = \text{N}_1\cdot 2,5 \Rightarrow 10 - 20x + 4 = \text{N}_1\cdot 2,5 \Rightarrow$$

$$\Rightarrow 14 - 20x = 2,5\text{N}_1 \Rightarrow \text{N}_1 = 5,6 = 8x$$

Ή πρέπει $\text{N}_1 \geq 0 \Rightarrow 5,6 - 8x \geq 0 \Rightarrow x \leq 0,7\,m$

$0 \leq x \leq 0,4\,m$ άρα δεν ανατρέπεται.